a smaller root-mean-square (rms) error than $p = k_\phi z^n$ In 46 cases of the 51, a second-degree polynomial $z = a_1 p + a_2 p^2$ gave a smaller root-mean-square (rms) error than $p = k_\phi z^n$ In 23 cases of the 51, a first-degree polynomial $z = a_1 p$ gave a better fit than $p = k_\phi z^n$—a surprising result, since, by taking $n = 1$, the two equations are identical in form The difference arises from the bias in the logarithmic transformation

Figure 1 illustrates a comparison for one of the soils tested The graph shows the original x-y data plot together with the curves predicted by two of the equations Table 1 is a tabulation of the rms error involved with each approximation for the data of Fig 1

Table 1 Tabulation of rms error associated with each equation

Equation	rms error, in $[(z - \bar{z})^2/N]^{1/2}$
$p = k_\phi z^n$	0 2802
$z = a_1 p$	0 0433
$z = a_1 p + a_2 p^2$	0 0383
$z = a_1 p + a_2 p^2 + a_3 p^3$	0 0186
$z = a_1 p + a_2 p^2 + a_3 p^3 + a_4 p^4$	0 0081
$z = a_1 p + a_2 p^2 + a_3 p^3 + a_4 p^4 + a_5 p^5$	0 0062
$z = a_1 p + a_2 p^2 + a_3 p^3 + a_4 p^4 + a_5 p^5 + a_6 p^6$	0 0043
$z = a_1 p + a_2 p^2 + a_3 p^3 + a_4 p^4 + a_5 p^5 + a_6 p^6 + a_7 p^7$	0 0043

Conclusions

Any polynomial of degree two or higher gives a better fit to the majority of the data considered than the power function now in use In addition, it is anticipated that a polynomial would give a better fit to data from nonhomogeneous soils, such as soft soil over hardpan The only practical question lies in where the polynomial should be truncated This truncation is simply a tradeoff between accuracy and complexity Two or possibly three terms would seem to be the appropriate compromise

One distinct disadvantage of the equation $p = k_\phi z^n$ is that k_ϕ has dimensions (force)/(length)$^{n+2}$ That is, its dimensions depend upon the shape of the pressure sinkage curve Thus, different scaling laws are required for different soils The polynomial does not suffer this disadvantage

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Proton Fluxes along Low-Acceleration Trajectories through the Van Allen Belt

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Introduction

THE radiation hazard of energetic protons trapped in the earth's magnetic field becomes an important consideration for manned, low-thrust, interplanetary vehicles because of the long period of time that such a vehicle must spend in the region near the earth The advantage of high-payload ratios attainable with low-thrust electrically propelled vehicles may be seriously offset by the necessity of increased weight to protect against Van Allen radiation

An estimate of the integrated dose received during a high-thrust lunar trajectory through the radiation belts behind various thicknesses of carbon shield has been made [1] Also, a method of computing total time-integrated proton flux for an arbitrary trajectory through the inner Van Allen belt has been formulated, and examples of integrated flux for circular orbits are offered in Ref 2 An approximate calculation of the upper and the lower limits of the integrated proton flux encountered during a low-thrust departure from an earth orbit was made[3] for a thrust-to-weight ratio of 10^{-4}

This paper presents an estimate of the time-integrated flux for trajectories having a constant acceleration in the range of 5×10^{-4} to 10^{-2} m/sec^2 and an inclination (angle between the vehicle orbit plane and the equatorial plane) in the range of 0° to 90°

Spatial Distribution of Flux

The distribution of the proton flux contours was assumed to be that given in Ref 3 and is reproduced in Fig 1 Flux contours of protons with energies greater than 40 Mev are plotted in the geomagnetic plane The maximum intensity is 4×10^4 protons/cm^2-sec This would be the actual distribution in space if the earth's magnetic field were a perfect earth-centered dipole field This, of course, is not the case The distribution is distorted to various degrees for different values of longitude The variations in latitude and altitude of the base point in the figure are given in Ref 3 It was assumed that every point in the distribution was rotated through the same angle and translated through the same distance as the base point for that particular longitude This assumption implies that the central region of the distorted field can be represented by the central portion of a dipole field

The approximation is better than the assumption that the earth's magnetic field can be represented by a displaced dipole field but is inferior to a description of the field based on a spherical harmonic analysis It is felt, however, that present-day uncertainties in the location of the upper-altitude contours in the dipole plane and in the temporal variations of the contour locations do not warrant the computer time required for low-thrust calculations based on an accurate field description

Vehicle Trajectory

The vehicle was assumed to have constant tangential acceleration throughout its passage through the belt The results are also applicable with little error to vehicles employing constant-thrust engines since the change in mass during this period is quite small for the specific impulses typical of electric propulsion

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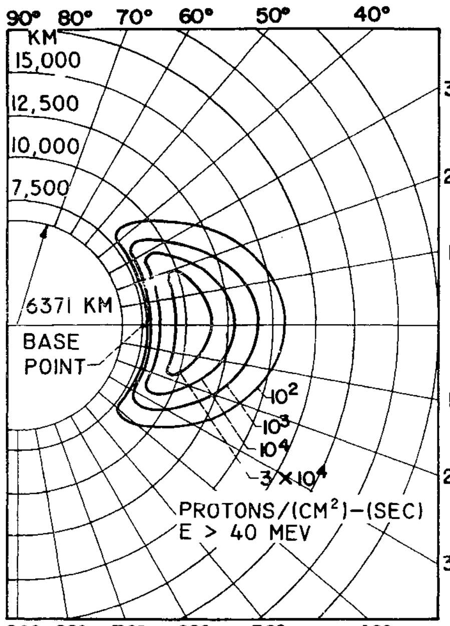

Fig 1 Proton distribution in the Van Allen belt plotted in the geomagnetic plane (reproduced from Ref 3)

For a given flux distribution in space, the integrated flux incident on a vehicle is a function of the acceleration a, the inclination of the spiral plane to the equator I, and, to some extent, the coordinates of the point at which the spiral is initiated: integrated flux $= f(a, I,$ starting point)

The integrated flux decreases with increasing acceleration because less time is spent in the region of the belts There is a practical upper limit on the acceleration of vehicles employing electric engines that results from the substantial weight of the necessary electrical power-generation equipment The effect of acceleration on integrated flux was investigated in the range of 5×10^{-4} to 10^{-2} m/sec^2

For a given set of conditions, the integrated flux would be a minimum for an inclination of 90° (polar spiral) Other mission considerations, however, might make a smaller value of inclination desirable The effect of inclination on the integrated flux was investigated throughout the range of possible inclinations

The altitude of the initial circular-orbit radius has negligible effect on the integrated flux as long as it is less than the altitude of the bottom of the belt A vehicle that initiates a spiral at a low altitude will eventually achieve the altitude of a vehicle that might start at a higher altitude The differences in the trajectories from that point on are negligible A best combination of longitude and latitude exists at which to initiate a spiral because of the radiation-free zones near the poles and the irregular shape of the belt about the earth For a large number of passes through the belt, as would be the case for a low-acceleration spiral, the advantage of starting at this point is negligible

Calculation Procedure

The integrated flux was calculated in terms of the equivalent number of hours a vehicle spends at the point of maximum intensity It was felt that this measure of the flux would have greater physical significance than would the total number of protons striking a square centimeter of area Equivalent hours, along with a set of dose-rate curves appropriate for the heart of the belt, can be used to estimate total radiation dose This procedure necessarily presumes that an invariant relative energy spectrum prevails throughout the belt, an assumption known to be untrue, but one that cannot be significantly improved upon with current information

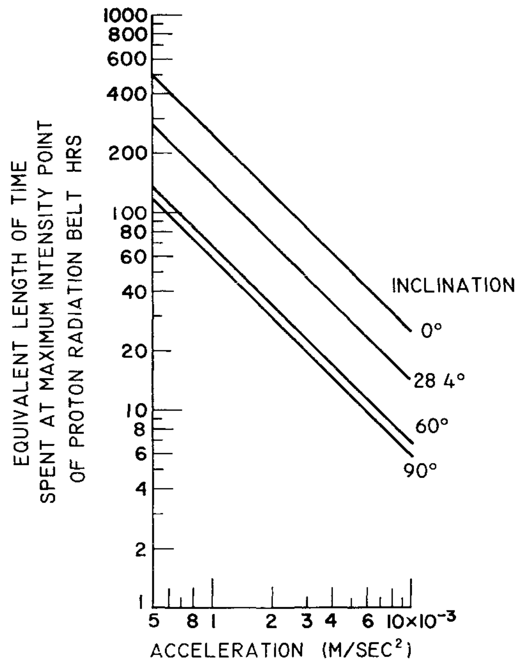

Fig 2 Effect of acceleration on integrated flux for various inclinations

A short IBM 7094 computer program was written to compute the equivalent time at maximum intensity The program accepts values of radius ratio ρ and central angle θ at various times t obtained from Ref 4 for a particular acceleration Values of ρ and θ are obtained at 2-min intervals by three-point interpolation These polar coordinates in a plane, inclined at an angle I with the equator, are transformed into geographic coordinates in a system that rotates with the earth Thus, the longitude, latitude, and altitude of the vehicle are obtained at each interval of time The geographic coordinates are then corrected for the variation of the position of the flux contours with longitude The corrected coordinates are used to obtain values of flux ratio (the ratio of flux at the point to the maximum flux) from a mathematical model of the distribution in the geomagnetic plane The upper contours of the distribution were approximated by elliptical segments and the lower contours by segments of a circle A smooth variation of intermediate contours was assumed Finally the values of flux ratio at 2-min intervals of time are numerically integrated by Simpson's rule The result is the equivalent length of time spent at the point of maximum intensity expressed in hours

Results

The effect of acceleration can be seen in Fig 2 Curves of equivalent hours at the heart of the belt vs acceleration are drawn for inclinations of 0°, 28 4°, 60°, and 90° An orbit plane of 28 4° would be achieved by launching a rocket due east from Cape Kennedy

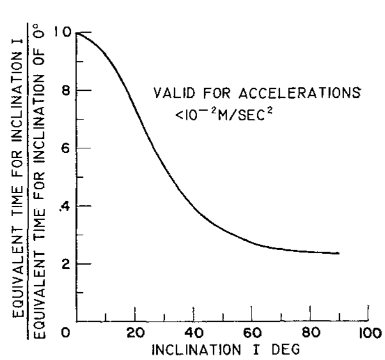

Fig 3 Effect of inclination on integrated flux

The effect of inclination on the integrated flux is shown in Fig 3 In this figure, the ratio of equivalent time for a particular inclination to the equivalent time for an equatorial spiral is plotted vs inclination A considerable gain can be realized by increasing the inclination a small amount in the range of low inclinations For a given acceleration, the integrated flux can be reduced by more than a factor of four by selection of a polar spiral rather than an equatorial spiral for a particular mission

For accelerations not exceeding 10^{-2} m/sec^2 (above which the starting point of the trajectory becomes an important parameter), the data of Figs 2 and 3 can be expressed by the simple relation, equivalent time (hr) $= K_1K_2/a$, where a is acceleration, m/sec^2; K_1 is 0 25 hr-m/sec^2, a constant obtained fiom Fig 2; and K_2 is the ratio of equivalent time for inclination I to equivalent time for inclination of 0°, obtained from Fig 3

As an example of the significance of the presented results, Fig 2 indicates that a spacecraft traveling with an acceleration of 5×10^{-3} m/sec^2 in a plane inclined 28 4° would accumulate a dose equivalent to a residence of 28 5 hr in the heart of the proton belt With the representative shielding data of Ref 5, this would correspond to a required shield density of 100 g/cm^2 of aluminum if the dose were to be limited to 30 rem from this source

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Simplified Calculation of the Jet-Damping Effects

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It has become traditional, since the work of Rosser, Newton, and Gross,[1] to calculate the jet damping of rockets as a sum of two effects: 1) the angular momentum flux carried by the exhaust, and 2) the effect of change of the moment of inertia Thomson[2] calculated the effect for a cluster of jets based on the same definitions

Fundamentally, the calculation of the jet damping involves the division of the rocket mass into two systems, namely, the rigid frame and the fuel moving relative to the frame The jet-damping effect can be defined as that part of the reaction of the fuel (i e, force and moment) on the rigid frame which arises from the angular velocity of the rocket in pitch or yaw In this note, the total effect will be calculated in one step by a method that leads to some simplifications, both conceptually and practically, and is applicable to both liquid and solid propellant rockets

The reaction of the propellant on the rocket will be determined from the equations of motion for the propellant in the fluid phase, which will be written down presently in a rotating frame For the purposes of jet-damping calculations, the fluid flow will be assumed inviscid, although not necessarily irrotational This is permissible as viscosity affects mainly the "basic" velocity distribution of the fuel flow in a nonrotating rocket, which can be approximated closely by an inviscid flow with proper vorticity distribution Thus, the effect of viscous stresses will be neglected, but the viscous effects on the basic flow distribution can be accounted for

A further simplification, which is also implied in all previous calculations, is the "quasi-steady" approach to be used presently By definition, jet-damping effects include reac tions due to angular velocity and not those arising from angular accelerations To exclude the latter effect, only the limit of very small accelerations will be considered, and the corresponding terms in the equations of motion will be omitted The angular velocity will be taken as a parameter varying so slowly that the calculations can be made "as if" the rotation were steady

No simplifying assumptions will be needed concerning the compressibility of the fluid It is also permissible that the total energy and the entropy vary from streamline to streamline

Let the equations of motion be written down in a rocket fixed Cartesian system (Fig 1) with the x axis parallel to the thrust line To fix ideas, only one component ω_z of the angular velocity of the rocket will be taken as nonzero; this means that the effect of roll (ω_x) is not the subject of the present investigation The equations of motion are as follows:

$$\left.\begin{aligned} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - 2\omega v &= -\frac{1}{\rho}\frac{\partial p}{\partial x} + \omega^2 x \\ u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + 2\omega u &= -\frac{1}{\rho}\frac{\partial p}{\partial y} + \omega^2 y \\ u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} &= -\frac{1}{\rho}\frac{\partial p}{\partial z} \end{aligned}\right\} \quad (1)$$

The aim is to calculate the effect of a small angular velocity ω on the pressure field p Accordingly, the effect of the "centrifugal" terms containing ω_z^2 will be neglected in comparison to the Coriolis terms proportional to ω

An elementary calculation, which provides all the essential results, can be made after making an apparently radical simplifying assumption on the flow field Let it be assumed that the fuel motion in the rocket is everywhere parallel to the x axis The fuel velocity u and the density ρ may, however,

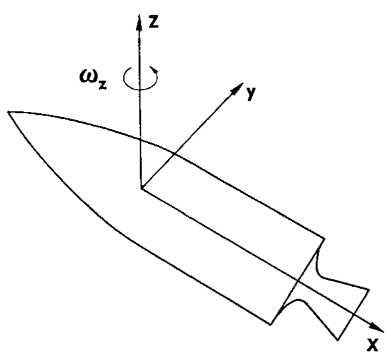

Fig 1 Coordinate system

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